Inclusion of Tactical Considerations for System-of-Systems Optimization of Torpedoes

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In the current torpedo design process, torpedoes are often designed independently from the tactics with which they are employed. This serial design process, of first developing tactics, then designing the torpedo, then re-developing tactics leads to torpedo designs that are sub-optimal when viewed from the greater system-of-systems perspective. This paper looks at the effects that tactics have on the design of torpedoes. It proposes a new paradigm, of simultaneous tactics development and torpedo design, and looks at the implications of various tactics on the optimal design of torpedo systems.

Nomenclature

AOU	=	area of uncertainty
a	=	sound absorption coefficient of seawater
BW	=	beam-width
DI	=	directivity index
DT	=	detection threshold
η_{search}	=	search-pattern efficiency
HP	=	motor horsepower
NL	=	background noise level
Otto	=	torpedo fuel
P_{hit}	=	probability of hit
R_{AOU}	=	radius of the area of uncertainty
SL	=	source level
Standoff	=	standoff distance of torpedo launcher
TOAD	=	Torpedo Optimization, Analysis, and Design Program
Vel	=	velocity

I. Introduction

In the current Navy environment of undersea weapons development, the engineering aspect of design is decoupled from the development of the tactics with which the weapon is employed. The current approach utilizes a group of intelligence experts and warfighters, drawing from knowledge that includes experience with previous weapons systems, wargaming scenarios, and threat assessments, who generate a preliminary set of 'desired' torpedo attributes. Warfare analysis groups then use complex engagement programs and tactical considerations to refine these preliminary attributes into point performance requirements for a future torpedo system, i.e., they specify a required maximum velocity, range, and turn rate. Torpedo designers then use engineering analysis tools to translate these requirements into feasible torpedo designs that meet the specified criteria.

Unfortunately, from the total systems perspective, this design paradigm may not produce optimal designs. For one, it leads to a situation in which the tactics with which a weapon is employed are developed independently from

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the weapon itself. The tactics are generally derived not from design knowledge of potential systems, but from experience with current operational systems, in conjunction with threat assessments, to develop required torpedo performance attributes to best defeat future threats. These performance attributes are set as requirements and passed down to torpedo designers, who then use their engineering models and available technologies to create a torpedo system that meets the analysts' specifications. Once this newer and more capable torpedo is introduced into service, the Fleet will often create a new set of tactics that best utilizes the capabilities of the new system. The tactics are therefore continuously developed and refined using a torpedo with static performance. This system of tactics development, then torpedo design, and then tactics re-development creates a never-ending cycle in which the weapon system is never truly optimized for the tactics with which it is employed. This lack of interaction between the warfare analyst and the weapon designer prevents the weapon system from reaching its greatest potential effectiveness. The current system is summarized in Figure 1.

Another drawback of this system is that weapon requirements are given to torpedo designers as a point condition, i.e., a specific speed and range are defined. These point conditions limit the torpedo designer to developing a torpedo that fits into a tightly constrained design space, curtailing design freedom and excluding potentially feasible designs that may better fulfill the mission with a different set of performance parameters.

Therefore, to truly optimize a weapon system, the tactical employment of the weapon and engagement models must be considered concurrently with the engineering analysis of the weapon. This concept introduces a new paradigm, in which mission analysis and weapon design are considered simultaneously. The inclusion of mission analysis, and the exploration of different combinations of tactics and performance, allows for the creation of an optimal weapon system. In addition, instead of designing to a rigid set of point requirements, the designer will now have the flexibility to adjust either torpedo performance attributes or the tactical employment to reach the required level of mission effectiveness, greatly expanding the design space and generating more freedom for the design process.

This paper looks at the implications that tactical variation have on the design and optimization of torpedo systems. It suggests that *simultaneously* developing tactics and designing the weapon will create weapon designs superior to those that have the tactics and the weapon independently optimized. This "system of systems" approach, looking at the torpedo in the larger context of the warfighting environment, is the best approach to providing the highest overall level of warfighting capability possible. The approach is illustrated in Figure 2.

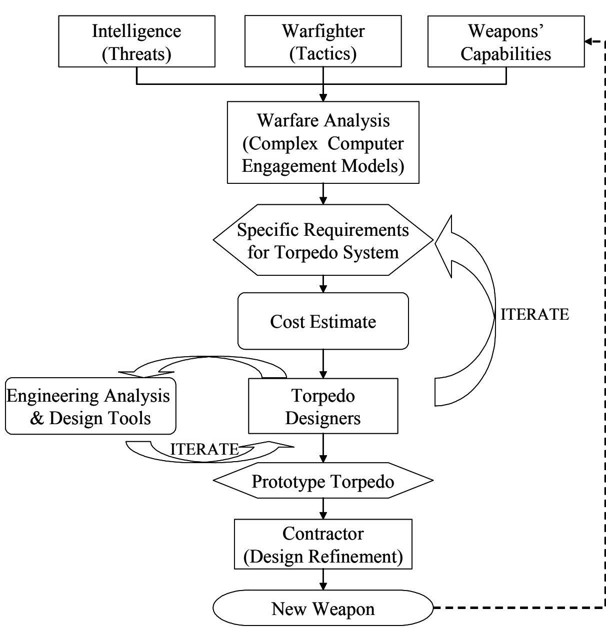


Figure 1: Current Torpedo Design Methodology (adapted from ref. 1)

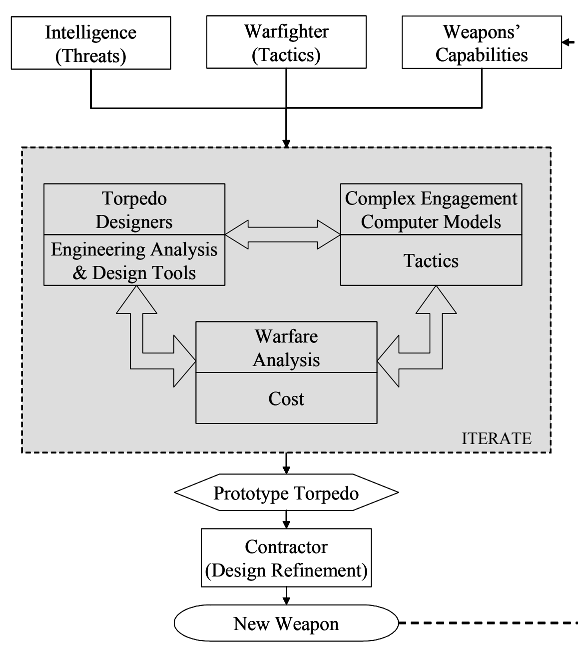


Figure 2: Proposed Torpedo Design Approach (adapted from ref. 1)

The original goal of this paper was to analyze the implementation of designing a torpedo system in conjunction with a modeling and simulation tool that accounted for submarine maneuvers, submarine tactics, the acoustic environment, and various methods for the tactical employment of torpedoes. Unfortunately, such analysis tools, in order to be accurate enough to be useful, are exceptionally complex and tend to be proprietary or classified in nature. As such, it was impossible to obtain the use of appropriate analysis tools for this paper. Instead, a simple torpedo sizing tool was developed, along with an accompanying engagement analysis tool that examined the likelihood of a torpedo system being able to locate and prosecute a target. Because of the limited modeling capabilities developed, a full system-of-systems optimization could not be performed, but, instead, an examination of the impact of the tactical situation at torpedo launch on the design of the torpedo and on the likelihood that it will hit the target. A more comprehensive analysis of the effects of tactics on weapon design would include the use of design parameters for the launching platforms (a.k.a. the submarine), creating a full system-of-systems environment, which will hopefully be implemented in future research.

II. Analysis Tools

In terms of operational environments, undersea warfare is almost completely characterized by the lack of knowledge concerning the location of enemy vessels. Thus, a large part of undersea warfare is concerned with the means and mechanisms of acquiring information about the target; essentially it is a combination of tactics and sonar capabilities[2]. In order to capture both the kinematics and the sonar capabilities of the torpedo, a simple torpedo design program was created, using response surface equations generated from the more advanced TOAD torpedo design and analysis program[3,4,5]. The new program was specifically designed to examine the implications of sonar performance on the overall capabilities of a fixed-length torpedo system. The program assumed a 240-inch long, 21-inch diameter torpedo, similar to today's heavyweight Mk-48 torpedo system[6]. The torpedo is divided into several sections, each sized independently, as described in Table I. The length of the nose section is a function of the directivity index and the beam-width of the sonar. In addition, the power requirement, or "hotel load", of the sonar is also a function of these parameters. The variation of hotel load as a function of sonar parameters is shown in Figure 3. The warhead is fixed as a 35 inch, 1,000-lb_m system. The motor provides power to the system, both for the propulsor and for the hotel load of the sonar. The length of the motor is a function of the shaft-horsepower required by the motor to provide sufficient thrust and power generation. The length is generated from a response surface equation developed from the more extensive TOAD analysis program. The back-end of the torpedo, including the afterbody, control fins, and propulsor, is fixed at a 30-inch length. Finally, the fuel section consumes the remaining length of the torpedo, sized so that the total length of the torpedo is always 240 inches.

Table I: Torpedo Sections

Section	Purpose	Size
Nose	Sonar and electronics	Function of DI and BW
Warhead	1,000 lb_m warhead	35 inches
Fuel	Fuel for motor	Remainder of 240" torpedo
Motor	Provide power to propulsor and sonar	Function of HP
Back-End	Rear of torpedo and propulsor	30 inches

In addition to estimating the sizes of individual torpedo sections, the torpedo analysis tool also calculates the detection range of the sonar, the total range of the torpedo, the search rate of the torpedo, and an estimate of the relative cost of the system. Two parameters are used to define the sonar system. Directivity index, in decibels, essentially tells the "goodness" of the sonar, or its ability to distinguish a target from background noise. Directivity index directly relates to detection range, as indicated in the formula below[7]. The beam-width defines the width of the sonar "beam". This width parameter is used to define the effective search area of the sonar. The hotel power drain of the sonar system is a function of both the beam-width and the directivity index. A graph of detection range versus directivity index is given in Figure 3, along with a plot of the power drain of the sonar.

$$20 \cdot \log_{10}(range) + a \cdot range = SL - NL + DI - DT$$

a = Absorption Coefficient of Seawater (0.00006 dB/m)[7]
SL = Source Level (25dB)
NL = Self-Noise Level (15 dB)
DT = Detection Threshold (03 dB)

DI = Directivity Index (input dB)

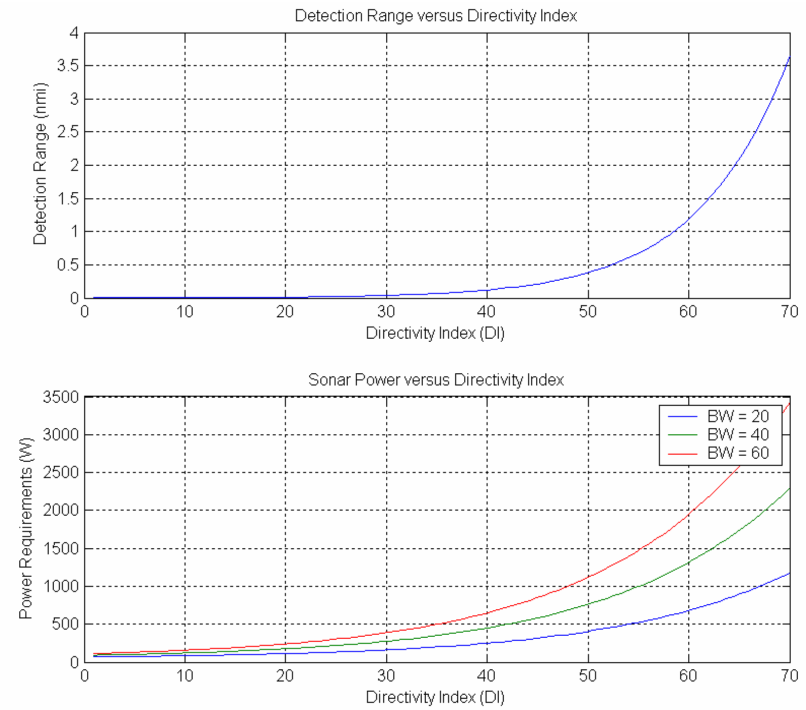


Figure 3: Comparison of Directivity Index with Detection Range and Hotel Load

The range of the torpedo is calculated by first looking at the drag of the vehicle, using the drag routines included in the TOAD analysis program. The power required to overcome the drag and the hotel load is used, along with the effective heating value (accounting for unburned fuel, fuel tank structure, auxiliary system volumes, and thermal engine efficiencies) for torpedo OTTO fuel used in the TOAD program (1.15 x 10^7 ft-lb_f/ft^3), to determine the endurance, and thus range, of the torpedo. Finally, the relative cost of the system is estimated. It is assumed that from the baseline torpedo there is an exponential price increase due to improving the sonar performance, plus a milder price increase for increasing the motor horsepower. These costs are entirely notional and are based upon the assumption that higher performance components, such as the engine and the sonar, will translate directly into greater costs. Figure 4 shows the relative performance and cost of the system when changing the torpedo design variables.

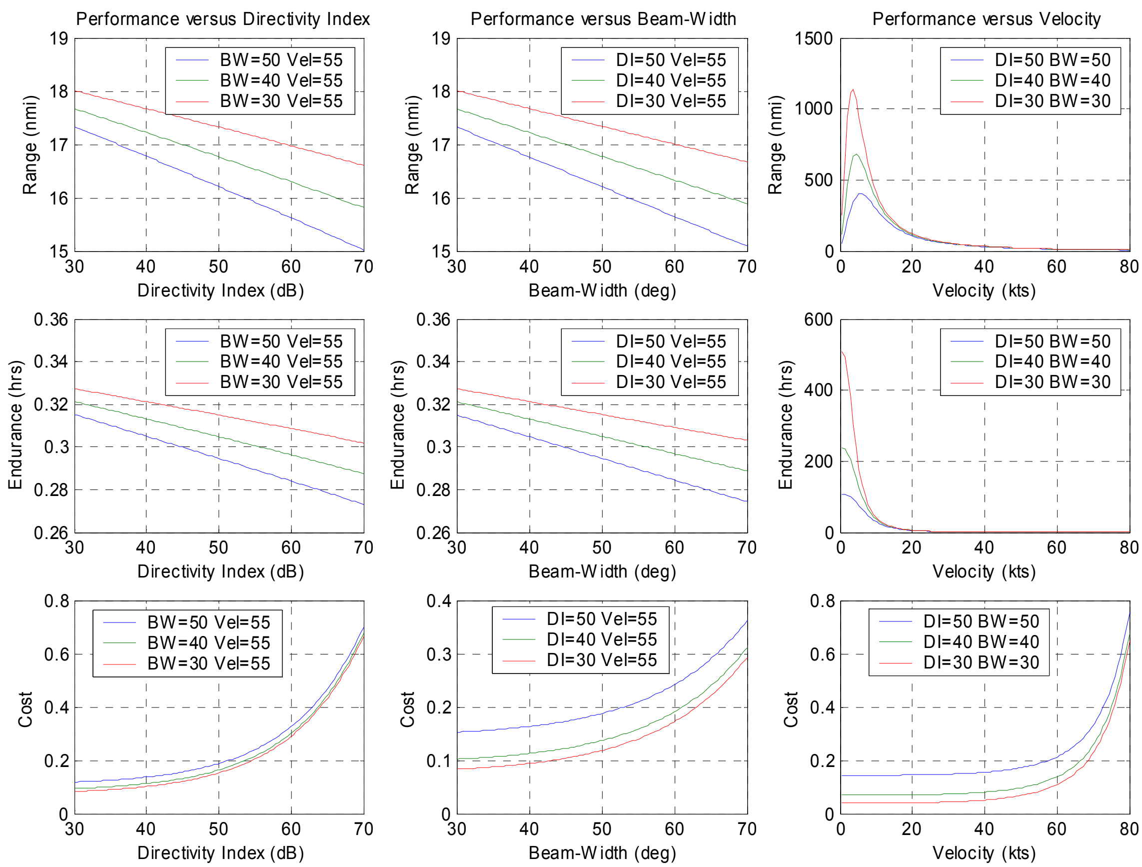
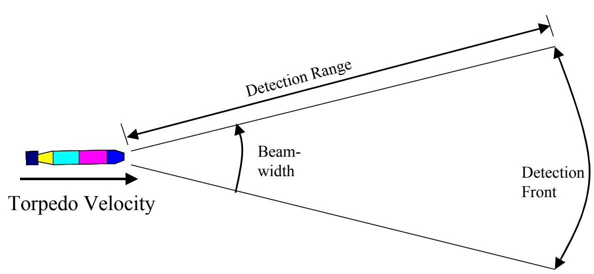


Figure 4: Range, Endurance, and Cost of Torpedo Systems

The search rate of the torpedo is calculated via the geometry shown in Figure 5. The search rate is calculated from the detection range (calculated from directivity index), the beam-width of the sonar, and the velocity of the torpedo. The search rate is essentially a calculation of how quickly the "search front" is moved through the water. The search rate assumes that the torpedo moves at a constant velocity and can "detect" anything that enters into the detection zone. In addition, the search rate assumes that a "perfect" search pattern is being executed (delineated in the formula by setting $\eta_{search}=1$). This perfect search pattern assumes that no portion of the search area is examined twice and that the threat submarine never doubles-back into a previously searched area. A less efficient search, with the torpedo overlapping previously searched areas, could be modeled by setting η_{search} to a value less than one.



$$SearchRate = \eta_{search} \cdot 2\pi \cdot DetectionRange \cdot ({BW_{torp}}/{360}) \cdot V_{torp}$$

Figure 5: Calculation of Torpedo Search Rate

Two tactical parameters are defined for this problem and are illustrated in Figure 6. The focus of these tactical parameters is not on the tactics, strategies, and maneuvers of the two submarines before firing, but instead focuses only on the tactical situation that the torpedo "sees" immediately after launch. Thus, the relevant torpedo information includes the distance to the target, or the required transit distance, and the area of uncertainty surrounding the target, which defines the size of the region within which the target is randomly located. An additional parameter is the velocity of the threat submarine, or, the rate at which the radius of uncertainty for the threat submarine is growing. The threat submarine is assumed to maintain a slow velocity to facilitate its hiding from the searching torpedo. Figure 7 demonstrates how an encounter develops over time.

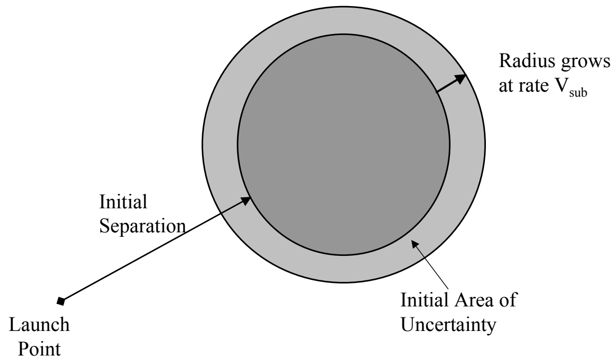


Figure 6: Tactics Parameters

Once launched, the torpedo transits the distance to the search area, with the required search area increasing during transit (at a rate equal to the velocity of the threat submarine). Once in the area of uncertainty, the torpedo begins its search pattern, searching at the constant rate defined in Figure 5. While the torpedo is searching for the target, the area of uncertainty continues to increase around the threat submarine. Figure 8 shows how the two relative areas change with time: the area that has successfully been searched by the torpedo and the area of uncertainty of the submarine. If it is assumed that the threat submarine is randomly positioned inside the area of uncertainty, then the ratio of these two areas defines the probability of the torpedo detecting the enemy submarine. Thus, the ratio of the current aggregate search area divided by the current area of uncertainty is defined as the probability of hit. Figure 8 also shows the time-varying probability of hit, which is calculated from the ratio of the two areas. The overall, or final, probability of hit is the maximum probability of hit attained during the encounter.

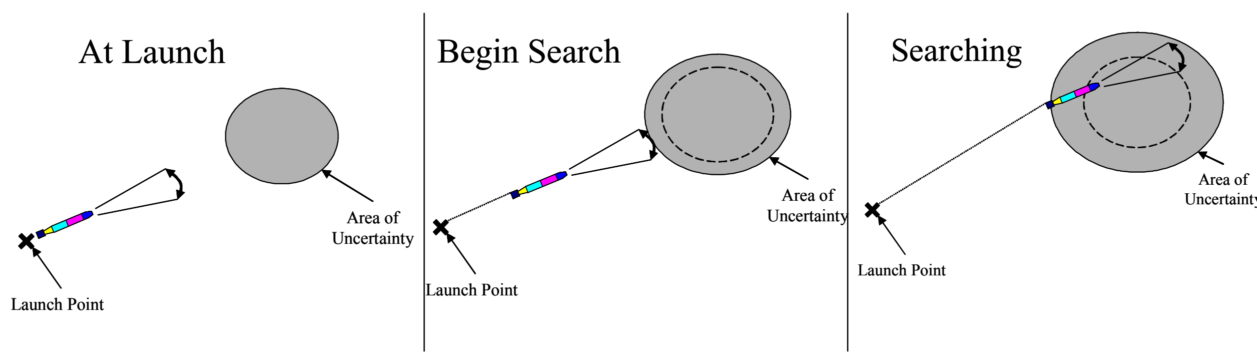


Figure 7: Encounter Dynamics as a Function of Time

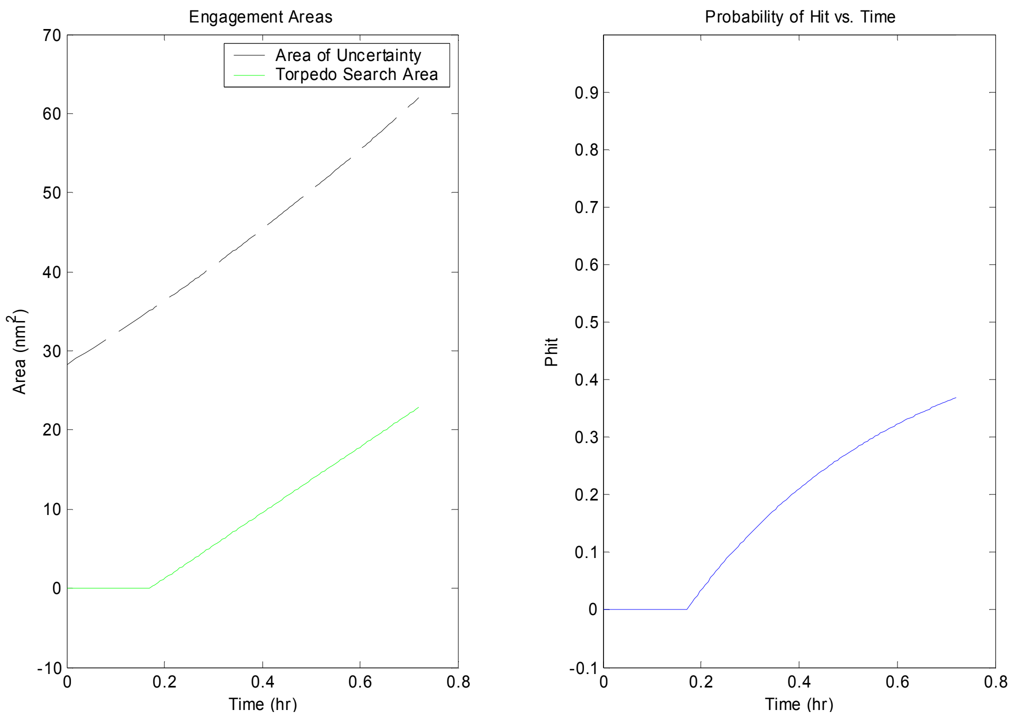


Figure 8: Time Variation of Area of Uncertainty, Torpedo Search Area, and Probability of Hit

The results from the torpedo analysis tool are linked with the inputs to the engagement analysis tool. The linkages are shown in Figure 9. Using this linkage between the two analysis tools, the torpedo inputs can be translated directly into a P_{hit} value. Since the torpedo cost is also calculated, the capabilities and cost for various torpedo systems can be compared under various operational scenarios. Table II summarizes the inputs and the outputs for the analysis system.

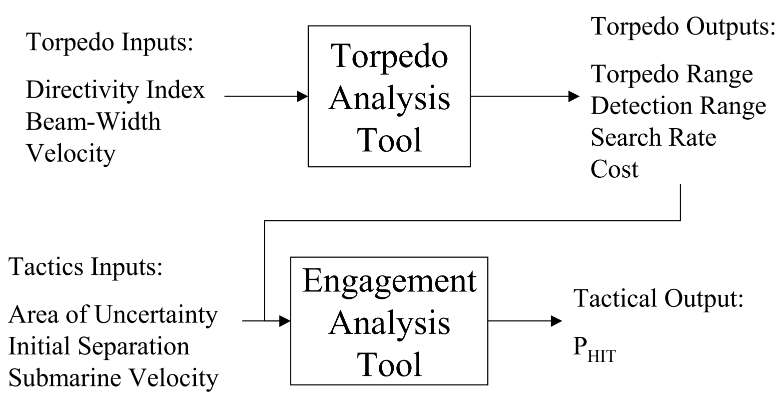


Figure 9: Layout of Torpedo Analysis Tools

Table II: Input and Output Parameters for the Problem

Parameter	Units	Description
Torpedo Input Parameters		
Directivity Index	dB	Effectiveness of sonar beam
Beam-Width	deg	Width of sonar beam
Velocity	kts	Torpedo velocity
Tactics Input Parameters		
Area of Uncertainty	nmi^2	Initial area of uncertainty for threat
Initial Separation	nmi	Initial separation to area of uncertainty
Threat Sub Vel.	kts	Threat submarine velocity
Torpedo Output Parameters		
Torpedo Range	nmi	Range of torpedo
Detection Range	nmi	Range that torpedo detects target
Search Rate	nmi^2/hr	Rate at which torpedo searches area
Cost	----	Estimated cost of system
Tactics Output Parameters		
P_{hit}	----	Probability of hitting target

Figure 10 summarizes the performance of various torpedo system designs. The figure shows the relative range, endurance, search rate, cost, and probability of hit for a torpedo system as a function of the physical attributes of the torpedo: directivity index, beam-width, and velocity. The figures provide some insight into the tradeoffs between the various torpedo attributes and overall performance. Figure 11 shows how varying the tactical situation changes the torpedo performance. The effects of changing standoff-distance, area of uncertainty, and the threat submarine velocity are apparent on the probability of hit. The figure also demonstrates the significant improvement in probability of hit with an improvement in the sonar system. Note that the torpedo has the best P_{hit} value when used with a standoff distance of zero and an infinitesimally small area of uncertainty. This situation corresponds to an "optimal" firing position, where the torpedo is dropped right on top of the enemy vessel and the torpedo knows the exact location of the enemy vessel.

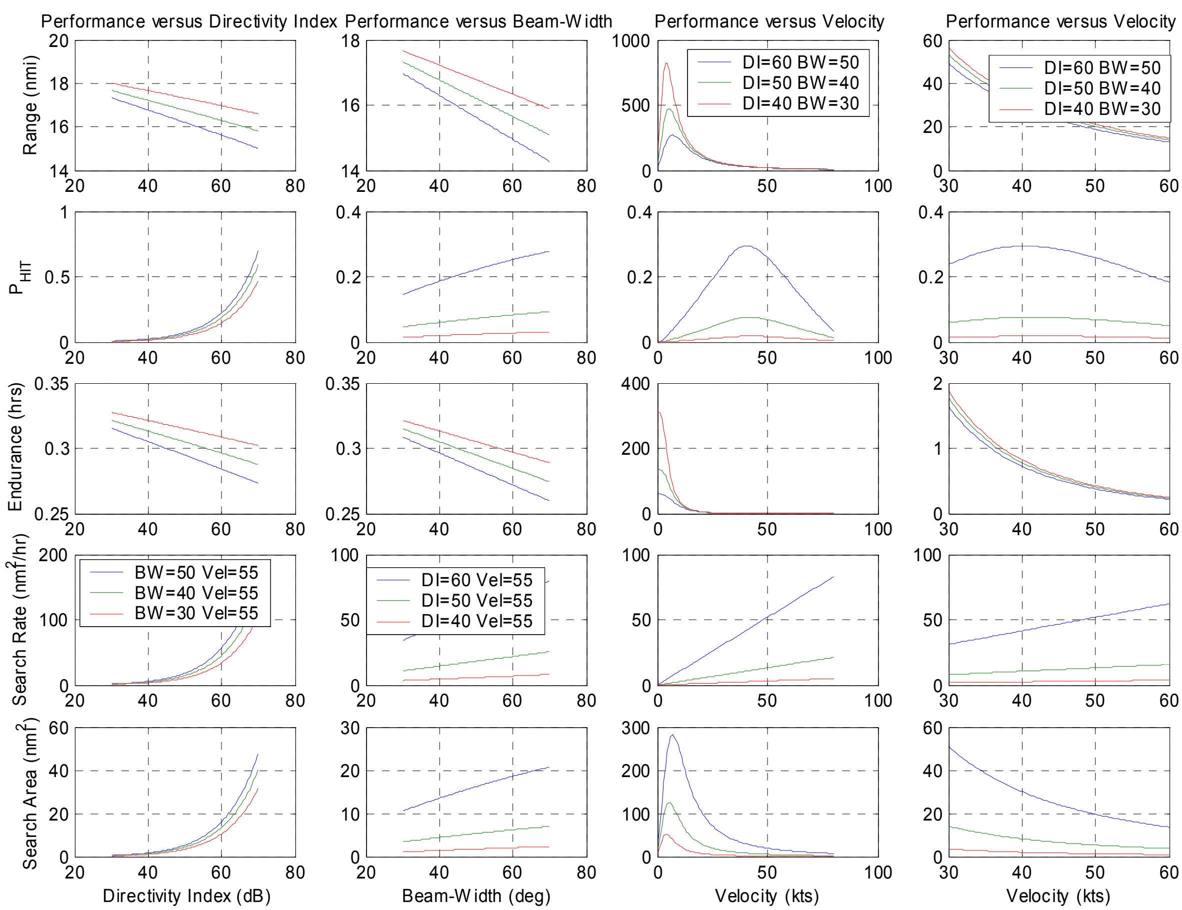


Figure 10: Torpedo System Performance

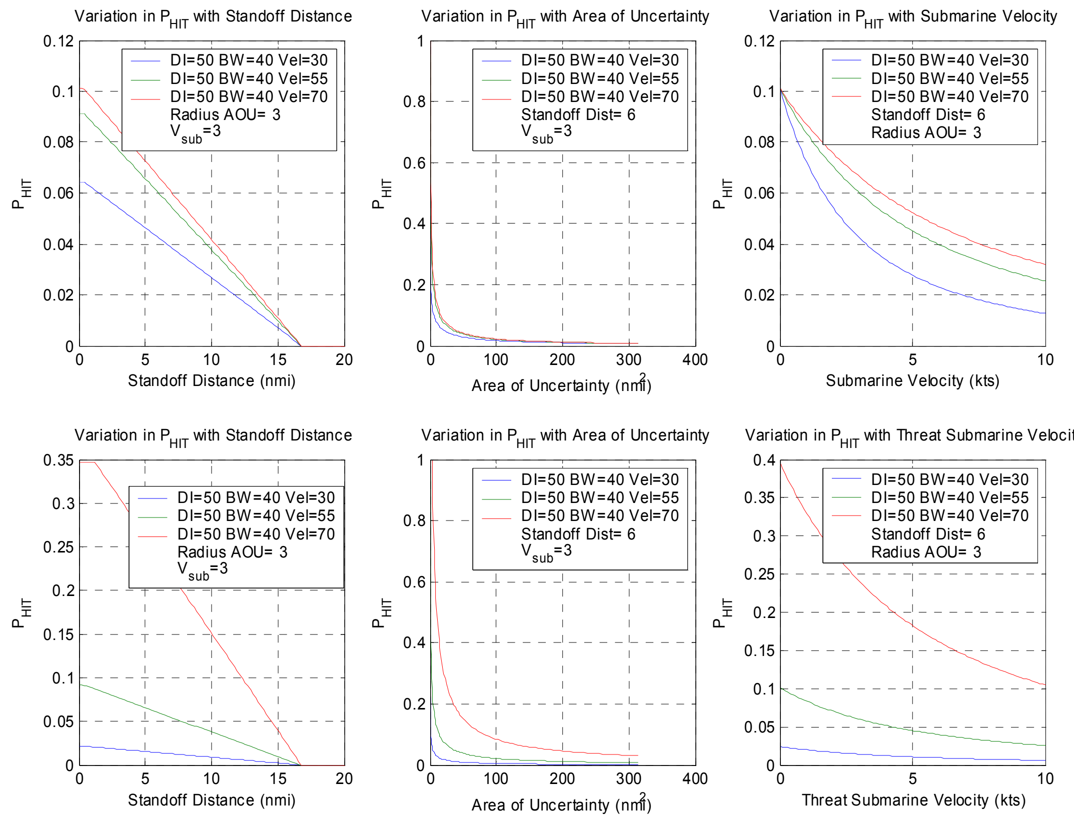


Figure 11: Variation in Torpedo Performance from Changing Tactical Environments

III. Optimization

Once the analysis tools were created and linked together, they were used in conjunction with the 'fmincon' optimizer in Matlab. The analysis tools, in conjunction with the optimizer, could then be used to find the lowest cost torpedo for a fixed price, or conversely, find the best performing torpedo for a fixed price. The graph in Figure 12 shows an example iteration history for a converged solution. The figure shows the convergence of a minimum cost torpedo with a constrained P_{hit} minimum of 0.8. In this example the optimizer behaves as expected, first meeting the P_{hit} constraint by driving up the cost, then working to reduce the cost while maintaining the minimum allowed P_{hit} value of 0.8.

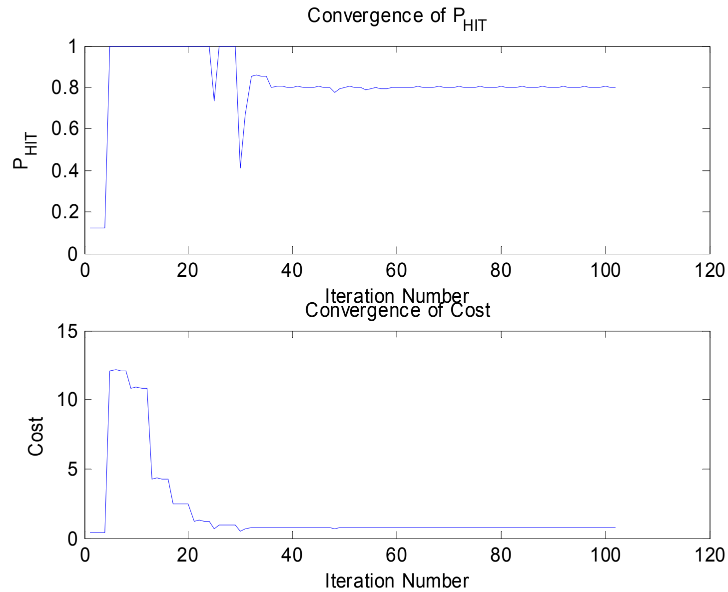


Figure 12: Convergence History for Minimum Cost Torpedo

Because the design space is multi-modal, multiple "starting locations" were used for the optimizer to guarantee that a global minimum, not just a local minimum, was found. Figure 13 shows the convergence history of a torpedo with a cost constraint of 0.5, where the optimizer is trying to maximize the value of P_{hit}. The convergence history of three independent starting points is shown in this figure.

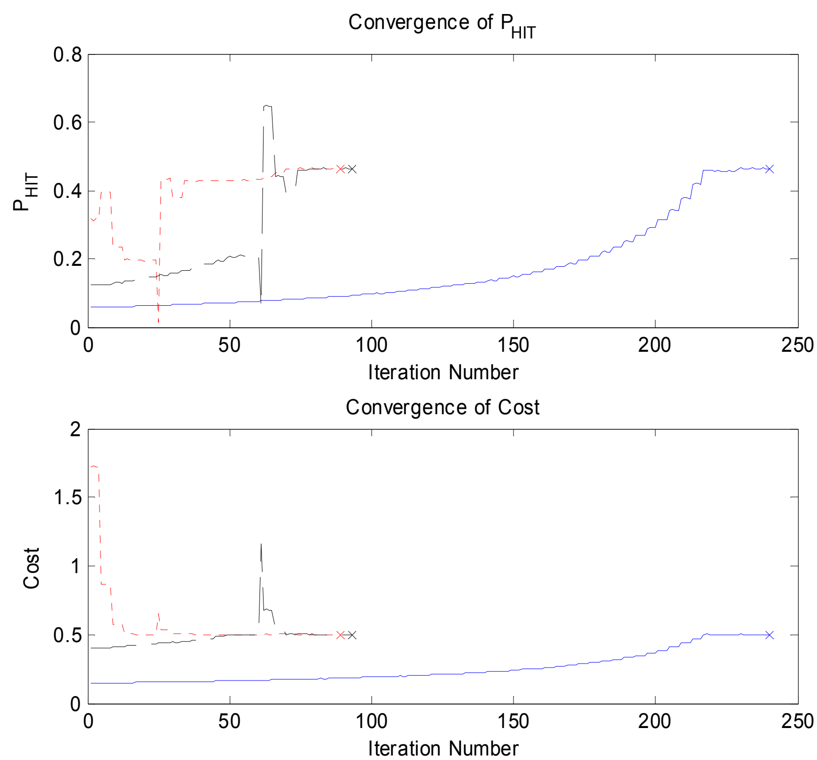


Figure 13: Convergence History of Maximum P_{hit} Torpedo with Multiple Start Points

Once the optimizer was successfully linked with the torpedo design programs, optimum torpedo designs were found for a range of constrained probability of hit values. The optimal, or lowest cost torpedo, was plotted for each probability of hit, and the results are shown in Figure 14 for three sets of tactical conditions. The figure verifies that, as more performance is required of the torpedo (in the form of a higher P_{hit} value), the system will be more costly. Note also that as the tactical environment worsens, as when the torpedo is launched further from the target area and more uncertainty exists about the location of the target, a much more expensive torpedo is required to meet the same level of probability of hit. Thus, a tradeoff is illustrated between the cost of the torpedo and the ability to launch the torpedo closer to the target. If the torpedo can be launched closer to the target, then a much lower-cost system will suffice.

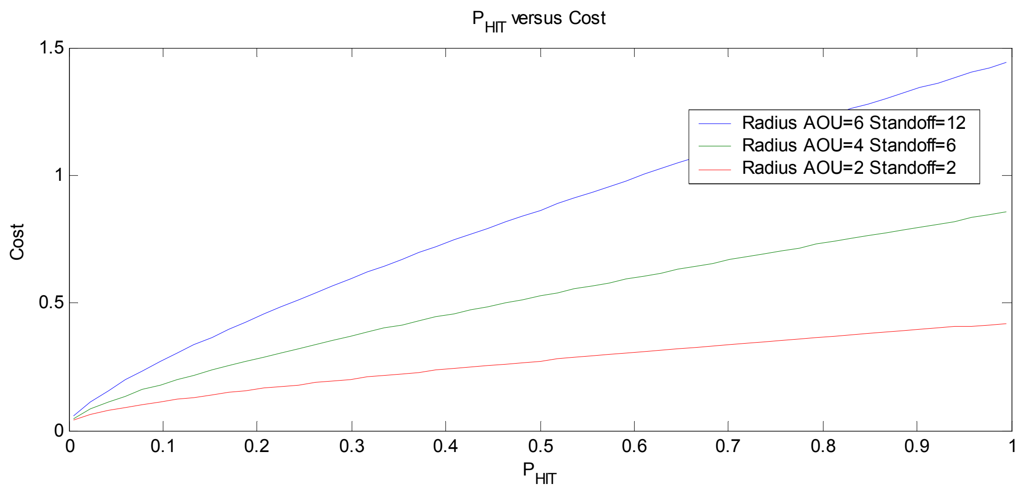


Figure 14: Lowest Cost Torpedo for Specified P_{hit}

Figure 15 gives more information about the optimal torpedoes for each probability of hit. The physical characteristics of each optimized torpedo: directivity index, beam-width, and velocity, are graphed, showing how the physical characteristics change for the optimal designs. Note that as the probability of hit requirement increases, both the directivity index and the beam-width increase, thus driving up the cost. Of interest is the fact that the velocity of the torpedo decreases as the required probability of success increases. This decrease indicates that it is more important to go slower and search for the target for a longer period of time than it is to cover an area quickly, but less effectively. Lastly, of note is the fact that when the torpedo is launched close to the target (2 nmi), with a small radius of uncertainty (2 nmi), a significantly faster torpedo with a smaller sonar is preferred. If the torpedo is launched this close to the target, it is apparently best to close to the target area quickly and forego the large, expensive sonar systems required of torpedoes that must search larger areas. The charts in Figure 15 show that the best torpedo for a given mission is a direct function of the tactical situation in which the torpedo is being operated. Thus, the tactics need to be developed simultaneously with the weapon, so that a weapon is always chosen that best fits the tactical environment.

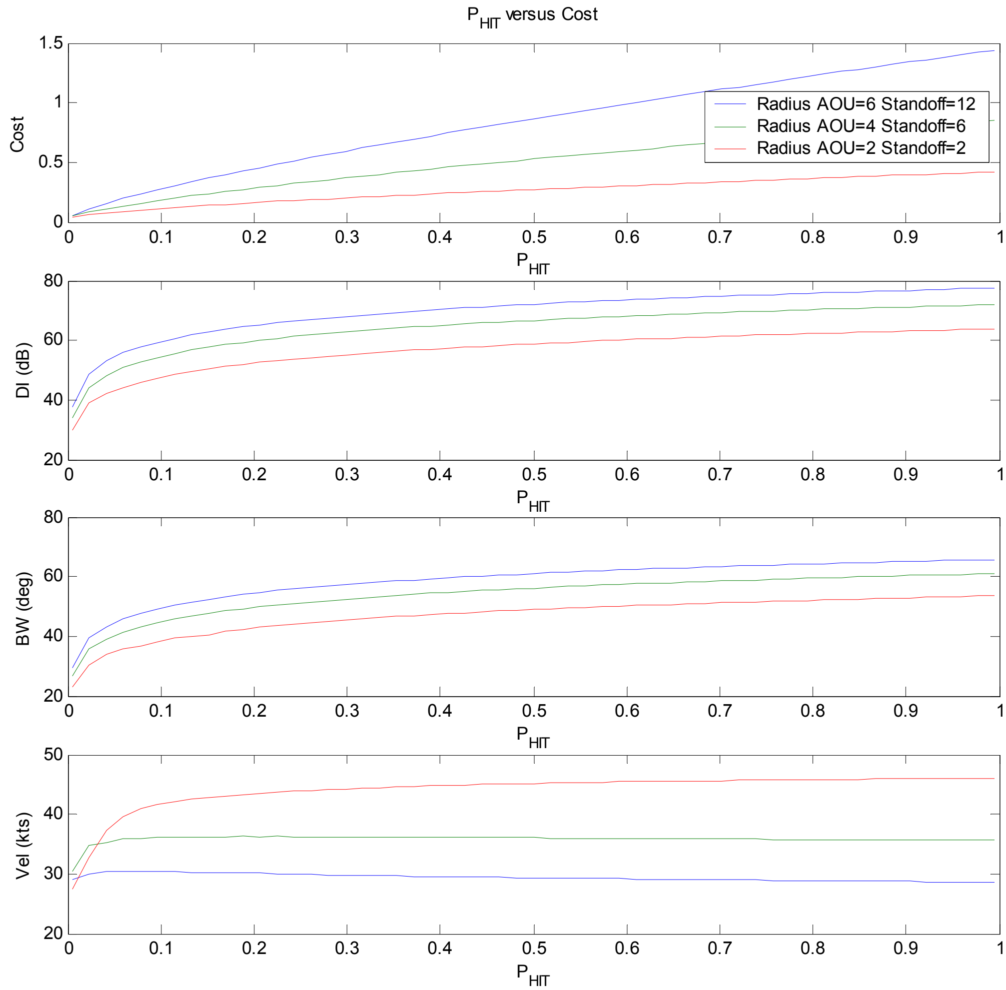


Figure 15: Illustration of Optimum Torpedo Attribute Variation with P_{hit}

Finally, the off-design performance of the torpedoes were tested for various tactical situations. First, a torpedo was optimized to provide the highest P_{hit} possible for a fixed cost of 0.5, in a tactical situation in which the radius of the area of uncertainty was 2.5 nmi and the standoff distance was 5 nmi. Table III shows the optimized torpedo for this tactical situation. At a cost value of 0.5, this torpedo was able to provide a probability of hit of 0.85 for the given tactical situation. An entire field of potential tactical situations was then run for this torpedo, to see how the effectiveness of the torpedo changed in various situations; the results are shown in Figure 16. Note that the torpedo performance degrades as it moves towards a longer standoff range and a larger radius of uncertainty. Not only does this represent a significantly degraded tactical environment, but since the plot is for a fixed torpedo that is not optimized for these situations, its performance should also be expected to decrease as the torpedo is being used in sub-optimal tactical situations. The right hand side of Figure 16 shows what happens when the torpedo is optimized for each set of tactical parameters. The torpedo is still constrained to meet the 0.5 cost requirement, however it is locally optimized for each tactical setting. Note that the same-cost torpedo performs significantly better when it is optimized for each tactical situation.

Table III: Optimized Torpedo System

Tactical Parameters		Optimal Torpedo Parameters		System Parameters	
Radius of Uncertainty	2.5 nmi	Directivity Index	66 dB	Cost	0.5
Standoff Distance	5 nmi	BeamWidth	55.4 deg	Phit	0.85
		Velocity	41.3 kts		
		Range	25.7 nmi		

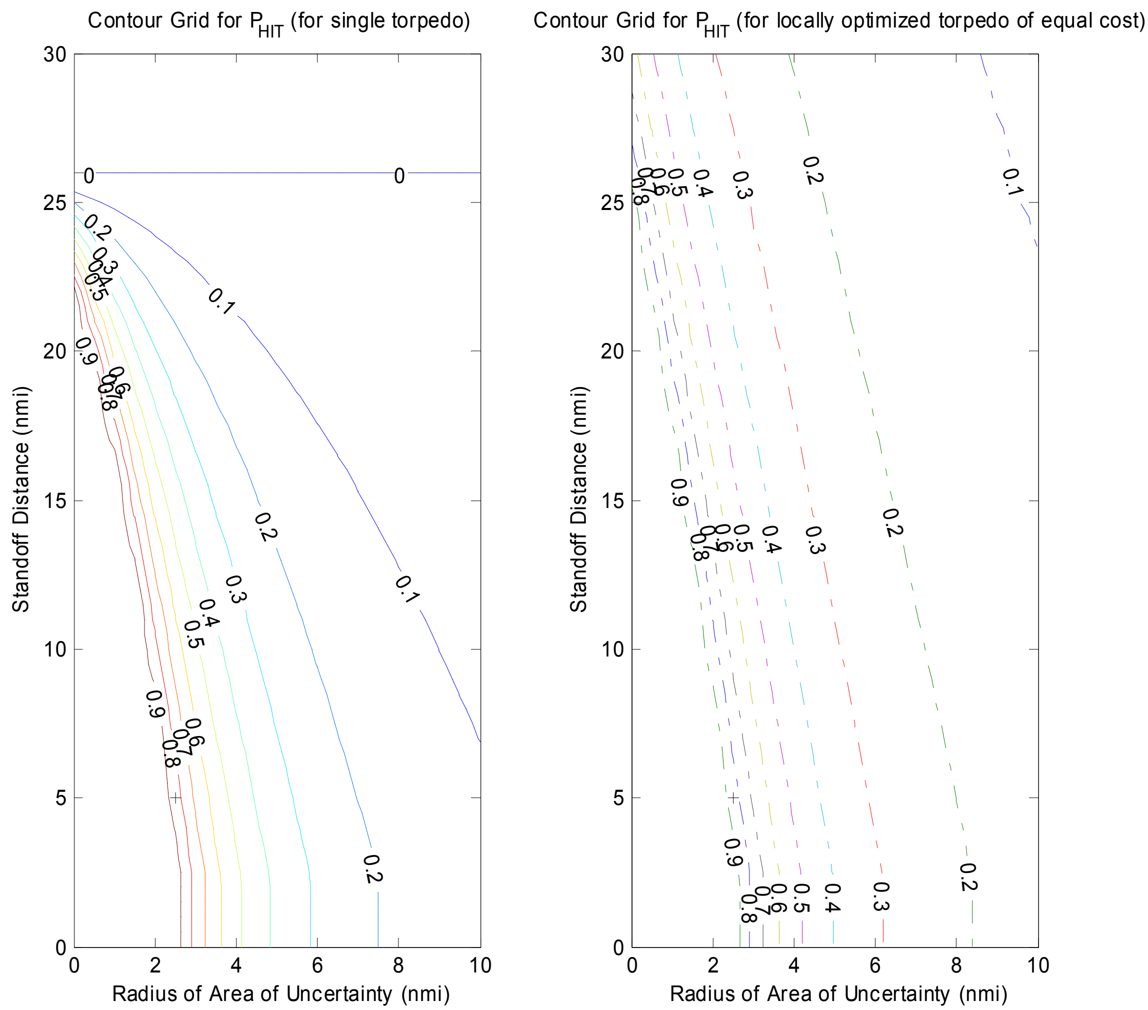


Figure 16: Contours for Varying Tactics

Figure 17 overlays the results of the fixed torpedo and the locally optimized torpedo. The dashed lines represent the locally optimized torpedo. Note that, near the "optimized" point, where the radius of uncertainty is 2.5 nmi and the standoff distance is 5 nmi, the results for the fixed and locally optimized torpedoes are identical. This is expected, as both torpedoes are essentially optimized for this region. However, the further away from this optimized region, or, the greater the change that occurs in the tactics, the greater the performance difference that exists between the two torpedoes. After altering the tactics only a small amount, the locally optimized torpedo begins to perform significantly better than the fixed, or singularly optimized system. Thus, the results indicate that there are significant advantages in optimizing the tactics simultaneously with the torpedo system, because the tactics need to be fully defined at the time of designing and optimizing the torpedo. Failing to have the tactics fully defined when constructing a torpedo system will potentially lead to a sub-optimal torpedo design.

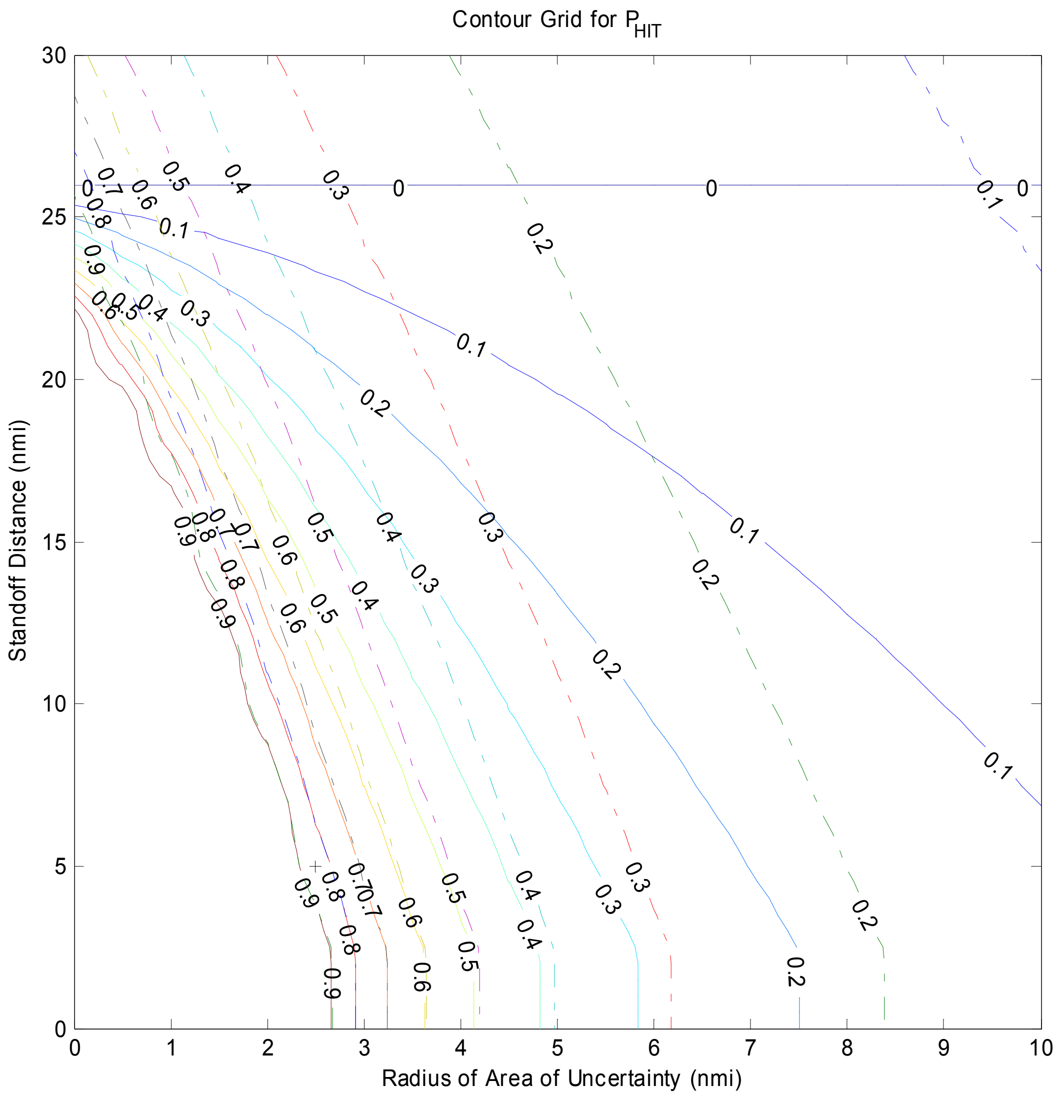


Figure 17: Overlay of Fixed Torpedo (solid line) and a Locally Optimized Torpedo (dashed line)

Figure 18 is similar to the comparison of the fixed and the locally optimized torpedo in Figure 17. However, in this case, the fixed torpedo is optimized for a different tactical situation: a radius of uncertainty of 5 nmi and a standoff distance of 15 nmi. The torpedo is again constrained to have a cost no greater than 0.5. The results again show that the locally optimized torpedo behaves similarly to the fixed design near the design point (marked with a +). But, again, there is significant improvement in the effectiveness of a locally optimized torpedo in tactical situations for which the fixed torpedo was not optimized.

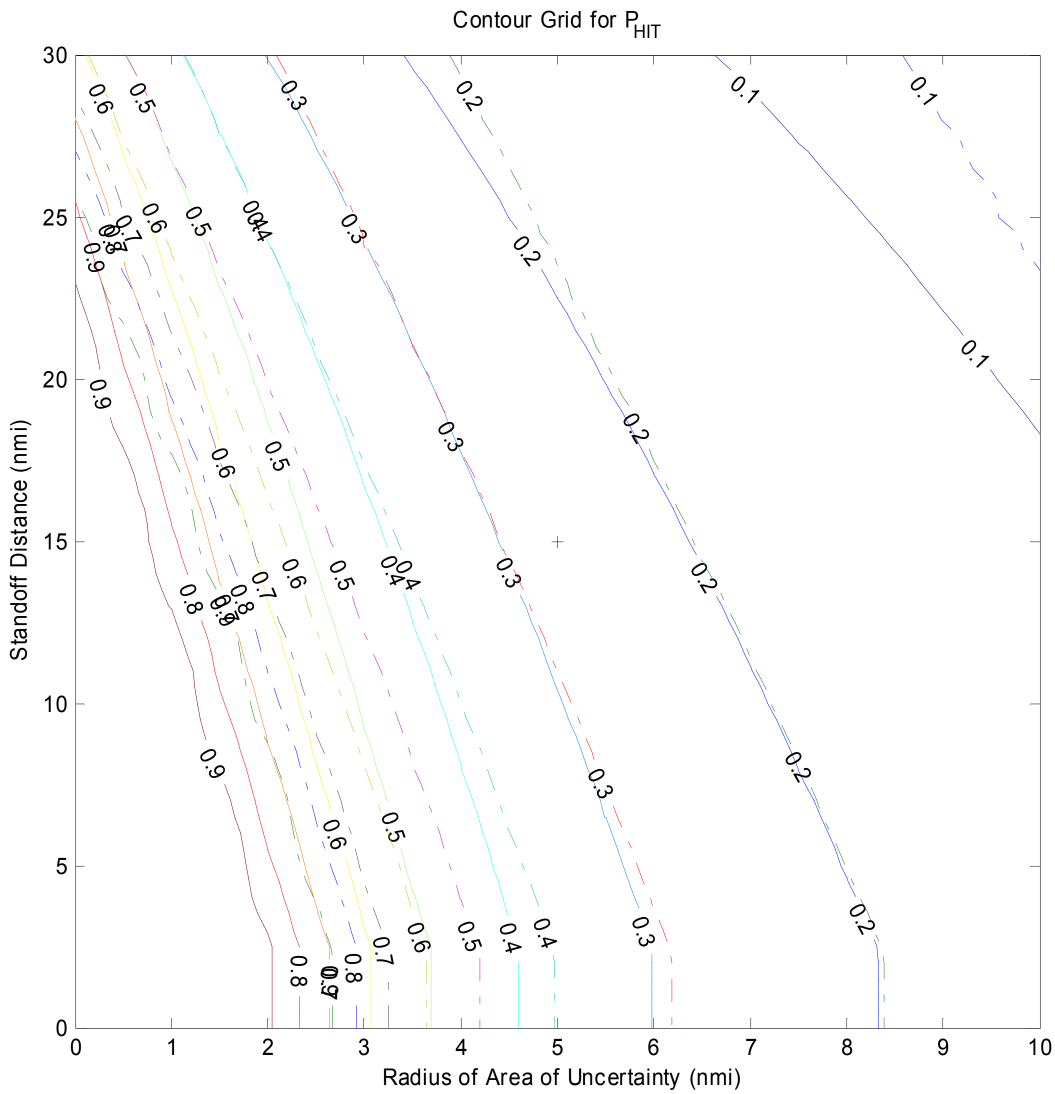


Figure 18: Overlay of Fixed (solid line) vs. Locally Optimized (dashed line) Torpedo

Figure 19 shows another example. This example is for a compromise torpedo design, one that attempts to have good performance at both extremes of the tactical space: a close launch with good target position data and a long-range launch with bad target position data. Note that this compromise torpedo does perform moderately well throughout the tactics space, but, the locally optimized torpedo outperforms the compromise torpedo at every single point in the space. Table IV shows a summary of the performance for the three fixed torpedoes shown in Figure 17 through Figure 19, along with a locally optimized torpedo. Note again that the locally optimized torpedo always outperforms the other torpedoes, regardless of the mission. Again, in order to achieve the best performance from a torpedo system, the tactics must be analyzed and created simultaneously with the design of the torpedo system.

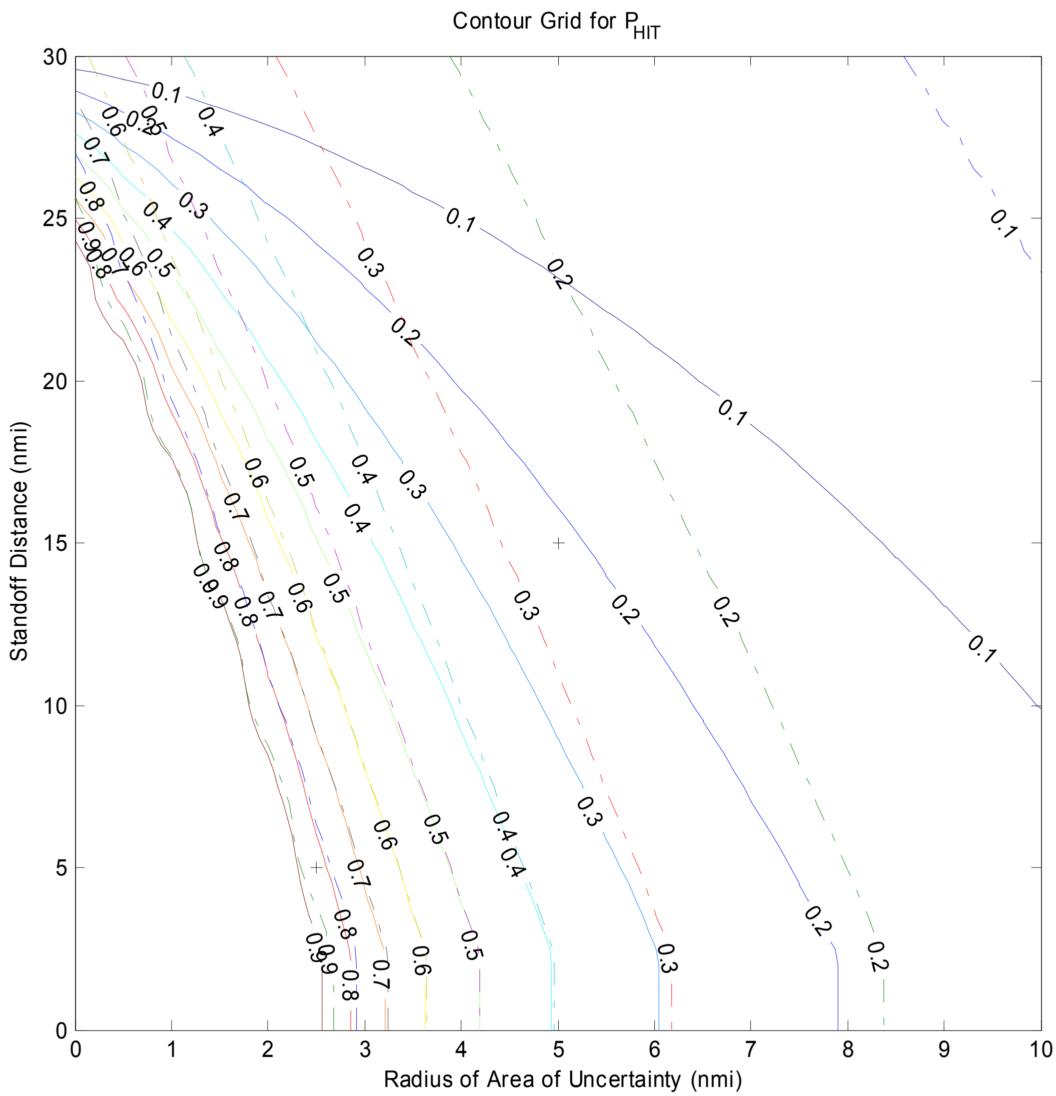


Figure 19: Compromise Torpedo (solid line) vs. Locally Optimized Torpedo (dashed line)

Table IV: Relative Performance of Various Torpedoes

			Close Range Torpedo	Long Range Torpedo	Compromise Torpedo	Locally Optimized Torpedo
		DI	66.0	66.5	66.1	varies
		BW	55.4	53.2	55.4	varies
		Velocity	41.4	30.1	37.9	varies
		Range	25.7	46.6	30.3	varies
Scenario 1 (Close In, Good Knowledge)	Radius of Uncertainty Standoff Distance	2.5 nmi 5 nmi	**0.852**	0.668	0.834	**0.852**
Scenario 2 (Long Range, Bad Knowledge)	Radius of Uncertainty Standoff Distance	5 nmi 15 nmi	0.174	**0.262**	0.215	**0.262**
Scenario 3 (Close In, Bad Knowledge)	Radius of Uncertainty Standoff Distance	5 nmi 5 nmi	0.335	0.345	0.356	**0.364**
Scenario 4 (Long Range, Good Knowledge)	Radius of Uncertainty Standoff Distance	2.5 nmi 15 nmi	0.442	0.486	0.504	**0.526**
Scenario 4 (Avg Range, Avg Knowledge)	Radius of Uncertainty Standoff Distance	3.75 nmi 10 nmi	0.384	0.404	0.417	**0.430**

IV. Conclusions

The results of this analysis show that the starting tactical situation of the torpedo, or the information available to the torpedo has when it is 'launched', has a significant impact on the performance of the system. Furthermore, the torpedo can be optimized so that its performance is maximized for any tactical scenario. However, if the torpedo is optimized for any single tactical situation, its performance will then be sub-optimal for other tactical situations. Thus, in order to get the most effective torpedo system, the tactics need to be defined and refined during the design and optimization of the torpedo system. By developing the submarine tactics simultaneous with the torpedo design, the torpedo design that best meets the tactical environments can be developed, thus ensuring the optimal, most cost-effective system. Therefore, this paper points to the need to design torpedoes in the context of the larger system (the submarine and tactical environment), thereby taking the "system of systems" approach. Only by designing the torpedo at such a level can truly optimal systems be created.

V. Acknowledgments

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